1. **Solution (a):** $Y = 0$, if $X = 0, 1, 2$. $P(Y = 1, X = 3) = 2p^3(1-p)^2$, $P(Y = 1, X = 4) = 4p^4(1-p)$, $P(Y = 1, X = 5) = p^5$.

   The rest of the probabilities can be computed using the distribution of X and X has $Bin(5, p)$ distribution.

   Solution (b): $P(X = 4|Y = 1) = \frac{4p(1-p)}{2-p^2}$

   $\square$

2. **Solution:** Let N be the number of empty poles when r flags of different colours are displayed randomly on n poles arranged in a row (here $r, n \in \mathbb{N}$ with $r \geq n$). Assume that there is no limitation on the number of flags on each pole. Let r flags be chosen randomly to be put on randomly chosen poles. If we follow this procedure, observe that the elementary outcomes have different probabilities. Let $A_{i,k}$ be the event that pole i is not chosen for flag k. $A_i = \cap_{k=1}^{r} A_{i,k}$ is the event that pole i is empty. By independence, $P(A_i) = \prod_{k=1}^{r} P(A_{i,k}) = (\frac{n-1}{n})^r$. Similary $P(A_i \cap A_j) = \prod_{k=1}^{r} P(A_{i,k} \cap A_{j,k}) = (\frac{n-2}{n})^r$. The number of empty poles, $N = \sum_{i=1}^{n} I_{A_i}$, is the sum of the indicator random variables of A_i's. Therefore,

$$E(N) = \sum_{i=1}^{n} P(A_i) = n\left(\frac{n-1}{n}\right)^r.$$

$$E(N^2) = \sum_{i,j=1}^{n} P(A_i \cap A_j) = n\left(\frac{n-1}{n}\right)^r + n(n-1)\left(\frac{n-2}{n}\right)^r.$$

   $\square$

3. **Solution:** Let M be the number of matches won by $B - I$ in the first stage and W be the number of matches won by $B - I$ in the second stage (out of M matches) Given M, W has binomial distribution $\mathbf{Bin}(M, 0.5)$. Therefore

$$E(s^W|M) = (0.5 * s + 0.5)^M.$$

   It is given that M has binomial distribution $\mathbf{Bin}(10, 0.5)$. Therefore

$$E(s^W) = E(E(s^Z|M)) = E(0.5 * s + 0.5)^M = (0.25 * s + 0.75)^{10}.$$

   Therefore W has binomial distribution $\mathbf{Bin}(10, 0.25)$. $\square$

4. **Solution:** Let F be the cumulative distribution function of a real valued random variable X. $F(x) = P(X \leq x)$. Since $P(X \in \mathbb{R}) = 1$, we have that $\sum_{k \in \mathbb{Z}} P(X \in (k-1, k]) = 1$ (by the countable additivity axiom of probability). Therefore we have that $\lim_{n \to -\infty} F(n) = \lim_{n \to -\infty} \sum_{k \leq n} P(X \in (k-1, k]) = 0$. From the monotonicity of F,$\lim_{x \to -\infty} F(x) = 0$. $\square$

5. **Solution (a):** Y is a non-negative random variable. $P(Y \leq y) = P(X^4 \leq y) = P(-y^{0.25} \leq X \leq y^{0.25}) = F(y^{0.25}) - F(-y^{0.25})$, where F is the cumulative distribution of X which has Normal distribution. Therefore, the probability density of Y is $\frac{1}{2\sqrt{2\pi}} y^{0.75} e^{-\frac{\sqrt{y}}{2}}$.

Solution (b): Stein's lemma. If X has $N(0,1)$ distribution, then $E(Xf(X)) = E(f'(X))$. This is done using integration by parts. $\int xe^{-\frac{x^2}{2}}dx = -e^{-\frac{x^2}{2}}$.
$E(Y^2) = E(X^8) = 7 * 5 * 3 * 1 = 105$.